

Tips, tricks and formulae on ratio and proportion

Ratio

- Ratio is comparison of two numbers, to find out how many times one number is greater than (or less than) the other number. It is to express one number as a fraction of other.

Ratio of two quantities a and b in same units, is the fraction $\frac{a}{b}$, where $b \neq 0$.

It is represented as a : b, where a (Numerator) is called as an antecedent and b (Denominator) is called as a consequent.

Multiplication or division of each term of a ratio by a constant does not affect the ratio.

Proportion

- Proportion is a special form of algebra equation, is used to compare ratios or make equivalent fractions.

The equality of two ratios is called as proportion, $\frac{a}{b} = \frac{c}{d}$ which is represented as, $a : b :: c : d$

a and d are called Extreme terms, b and c are called mean terms.

- Direct and Inverse proportion,

I. We say that a is directly proportional to b, $a \propto b$; if $a = kb$ for some constant k

II. We say that a is inversely proportional to b, $a \propto \frac{1}{b}$; if $a = \frac{k}{b}$ for some constant k,

Things to remember

- Compounded ratio of (a : b), (c : d) is (ac : bd), It is the ratio of product of first terms in every ratio to the product of second term in every ratio.

- For a ratio of a : b,

✓ Duplicate ratio of a : b = $a^2 : b^2$

✓ Sub-duplicate ratio of a : b = $\sqrt{a} : \sqrt{b}$

✓ Triplicate ratio of a : b = $a^3 : b^3$

✓ Sub- triplicate ratio of a : b = $\sqrt[3]{a} : \sqrt[3]{b}$

✓ Reciprocal of a ratio a : b = b : a

- If a, b, c and d are in proportion, $\frac{a}{b} = \frac{c}{d}$, $a:b::c:d$ then,

Product of extreme terms = Product of mean terms, i.e. $ad = bc$.

- For a proportion of $a:b::c:d$,
 - ✓ d is the fourth proportional of a, b, c
 - ✓ c is called third proportional to a, b
 - ✓ Mean proportional between a and b is \sqrt{ab}
 - ✓ Invertendo of $\frac{a}{b} = \frac{c}{d}$ is $\frac{b}{a} = \frac{d}{c}$
 - ✓ Alternendo of $\frac{a}{b} = \frac{c}{d}$ is $\frac{a}{c} = \frac{b}{d}$
 - ✓ Componendo of $\frac{a}{b} = \frac{c}{d}$ is $\frac{(a+b)}{b} = \frac{(c+d)}{d}$
 - ✓ Dividendo of $\frac{a}{b} = \frac{c}{d}$ is $\frac{(a-b)}{b} = \frac{(c-d)}{d}$
 - ✓ Componendo and Dividendo of $\frac{a}{b} = \frac{c}{d}$ is $\frac{(a+b)}{(a-b)} = \frac{(c+d)}{(c-d)}$
- If $\frac{a}{b} = \frac{c}{d} = k$, then $k = \frac{(a+b)}{(c+d)}$

Sol:

$$\frac{a}{b} = \frac{c}{d} = k$$

$$\Rightarrow \frac{a}{b} = k, \frac{c}{d} = k,$$

Write a in terms of b and c in terms of d

$$\Rightarrow a = kb, c = kd,$$

Substitute the above values of a and b in $k = \frac{(a+b)}{(c+d)}$ and verify,

$$\Rightarrow k = \frac{(a+b)}{(c+d)} = \frac{(kb+kd)}{(b+d)} = \frac{k(b+d)}{(b+d)} = k$$

Therefore, for $\frac{a}{b} = \frac{c}{d} = k$, then $k = \frac{(a+b)}{(c+d)}$

- The ratio $\frac{a}{b} > 1$ is called as ratio of greater inequality,

$$\frac{(a+k)}{(b+k)} > \frac{a}{b} \text{ and } \frac{(a-k)}{(b-k)} > \frac{a}{b}, \text{ for } k > 0$$

If the ratio $\frac{a}{b} < 1$, then

$$\frac{(a+k)}{(b+k)} < \frac{a}{b} \text{ and } \frac{(a-k)}{(b-k)} < \frac{a}{b}, \text{ for } k > 0$$

Formulae to remember

1. If a, b, c and d are in proportion then,

Product of extreme terms = Product of mean terms, i.e. $ad = bc$.

2. If $\frac{a}{b} = \frac{c}{d} = k$, then $k = \frac{(a+b)}{(c+d)}$

Model questions

- A quantity q divided in the ratio of $a : b$, then each part of the quantity is

I. The first part is $\frac{a}{(a+b)} * q$

II. The second part is $\frac{b}{(a+b)} * q$

- If a number n is added to a ratio $a : b$ such that the ratio becomes $p : q$, then $n = \frac{(aq - bp)}{(p - q)}$

Sol:

The number n is added to the ratio $a : b$

$$\begin{aligned} \Rightarrow \frac{a+n}{b+n} &= \frac{p}{q} \\ \Rightarrow q(a+n) &= p(b+n) \\ \Rightarrow qa + qn &= pb + pn \\ \Rightarrow qn - pn &= pb - qa \\ \Rightarrow n(q - p) &= pb - qa \\ \Rightarrow n &= \frac{(pb - qa)}{(q - p)} \end{aligned}$$

Therefore, the number n is $\frac{(pb - qa)}{(q - p)}$

- If $a : b = x : y$ and $b : c = p : q$, then $a : b : c = px : py : yq$

Sol:

$a : b = x : y$ and $b : c = p : q$

$\frac{a}{b} = \frac{x}{y}$, write a in terms of b , which is $\Rightarrow a = b * \frac{x}{y}$

$\frac{b}{c} = \frac{p}{q}$, write c in terms of b , which is $\Rightarrow c = b * \frac{q}{p}$

Now, the ratio of $a : b : c = \frac{bx}{y} : b : \frac{bq}{p}$

Multiplying or division of a constant on all the terms of ratio, doesn't affect the ratio of $a : b : c$,

Multiply y to all the terms in ratio, such that $a : b : c = y * \frac{x}{y} : y * 1 : y * \frac{q}{p} = x : y : \frac{yq}{p}$

Multiply p to all the terms in ratio, such that $a : b : c = p * x : p * y : p * \frac{yq}{p} = px : py : yq$

Therefore, the ratio $a : b : c = px : py : yq$

- The ratio between milk and water is $a : b$, the total quantity is Q . Then find the amount of water x to be mixed so that the ratio becomes $c : d$.

Sol:

The ratio between water and milk is $a : b$,

Let the amount of milk = ak

Let the amount of water = bk , where k is a constant

Total quantity = Q , which is equal to $(ak + bk)$

The amount of water to be mixed = x

$$Q = ak + bk = K(a + b) \Rightarrow K = \frac{Q}{(a + b)}$$

So,

$$\begin{aligned} \Rightarrow \frac{ak}{bk + x} &= \frac{c}{d} \\ \Rightarrow akd &= c(bk + x) \\ \Rightarrow akd &= cbk + cx \\ \Rightarrow akd - cbk &= cx \\ \Rightarrow \frac{k(ad - cb)}{c} &= x \end{aligned}$$

Substitute k in the above equation, such that $x = \frac{Q(ad - cb)}{c(a + b)}$

Points to remember

1. A quantity q divided in the ratio of $a : b$, then each part of the quantity is

I. The first part is $\frac{a}{(a+b)} * q$

II. The second part is $\frac{b}{(a+b)} * q$

2. If a number n is added to a ratio $a : b$ such that the ratio becomes $p : q$, then $n = \frac{(aq - bp)}{(p - q)}$

3. If $a : b = x : y$ and $b : c = p : q$, then $a : b : c = px : py : yq$

4. The ratio between milk and water is $a : b$, the total quantity is Q . Then, the amount of water x to be mixed so that the ratio becomes $c : d$ is $x = \frac{q(ad - cb)}{c(a + b)}$